Measurements of the Effect of Suspended Small Particles on Turbulent Mixing in Pipe Flow

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In chemical industrial operations, slurry transport is indispensable. Turbulent mixing in solid-liquid suspensions becomes even more important when chemical reactions are carried out during slurry pipe flows. This paper deals with the effect of suspended solid particles on eddy diffusivity in turbulent liquid in pipe flow.

After a tracer is introduced at the center of a circular pipe, at a perpendicular plane downstream from the source, the spread of material is characterized by the lateral component of the mean-squared displacement of the material from its center of mass. The rate of spread of the tracer can be determined by measuring the concentration across the diameter downstream from the point source. If the concentration profile is assumed Gaussian and longitudinal diffusion negligible, the mean square of lateral displacement $\overline{x^2}$ can be determined from a semilog plot between the local concentration of the tracer and r^2.

For a homogeneous turbulent field, following the analysis of Taylor (1935), one can arrive at Equation (1):

$$\overline{x^2} = 2u'^2 \left[\int_0^t (t - h)\, R_L(h)\, dh \right] \qquad (1)$$

Various functional forms for the correlation coefficient $R_L(h)$ have been suggested in the literature. With

$$R_L(h) \simeq e^{-Ah} \qquad (2)$$

Equation (1) will reduce to the following, for large times:

$$\lim_{t>>0} \overline{x^2} = 2u'^2 L_h t - 2u'^2 L_h^2 \qquad (3)$$

From Equation (3) and the experimental data of $\overline{x_1^2}$ at different t, both L_h and u'^2 can be determined. Also

$$\lim_{t>>0} \frac{1}{2} \frac{d\overline{x^2}}{dt} = (u'^2 L_h) = D_E \qquad (4)$$

where D_E is the eddy diffusivity.

In this work, L_h, u'^2, and D_E are determined for flow of starch slurries in circular pipes. In examining the effect of starch particles on mixing in pipe flow, the micromixing time constant t_E is also determined. According to Corrsin (1957, 1964)

$$\frac{c'^2}{c_0'^2} = I_c = e^{-t/t_E} \qquad (5)$$

In Equation (5), c'^2 is the mean square of fluctuating concentration of the tracer which can be measured experimentally. Its ratio to that at time zero is the fluctuation intensity I_c.

EXPERIMENTS

A specially designed, temperature compensated, electrical conductivity probe was used to measure the concentration of a salt tracer injected into the liquid in turbulent pipe flow at its center. The main test section was the central 3 m portion of a 9 m long, 0.057 m Plexiglas pipe. The extra length on both ends was for eliminating entrance or exit effects.

Perhaps a special mention should be made about the design of a temperature compensating feature of the probe and the accessory circuit. In brief, the salt solution and a thermistor have a negative temperature coefficient of resistance. Thus, a reverse output signal of the thermister when connected in series with the conductivity probe will have a compensating effect. With the incorporation of the temperature compensating circuit, a single master calibration was sufficient for concentration readings at different temperatures.

The flow characteristics of starch slurries were determined in a capillary viscometer and the results reported elsewhere. Corn starch particles have an average particle size of 1.02×10^{-7} m. Their specific gravity is 1.486. For concentrations below 5% (by weight), starch slurries behave Newtonian.

RESULTS

The salt solution was introduced at the center of the pipe through a small tubing. At a plane downstream from the source, the concentration was measured by the conductivity probe across the diameter of the pipe over a period of time. The spread of material was characterized by the lateral component of the mean-squared displacement of the material from its center of mass. Typical recorded conductivity measurements are reproduced in Figure 1. From such recorded results, $\overline{x^2}$ values were determined by plots of $\ln \overline{C}$ vs. r^2.

With the determined $\overline{x^2}$ at different time, values of L_h and u'^2 were determined from the slope and the intercept of linear plots according to Equation (3). Table 1 gives the results of L_h at three different Reynolds numbers with

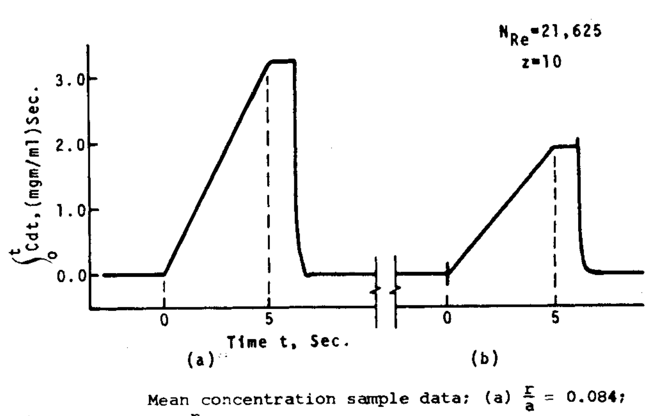

Fig. 1. Recorded mean concentration data at two different r values. Reynolds number = 21 625.

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Table 1. Lagrangian Time Scale Parameter, L_h in Seconds

Starch slurry concentration	Reynolds number, N_{Re} 16 220	21 625	27 035
0.5% (by wt.)	0.08477	0.0617	0.0546
1.0%	0.0816	0.0541	0.0584
2.5%	—	0.049	0.0508

Table 2. Time Constant for Micromixing*

Concentration of starch slurry	N_{Re}	$t_E(s)$
0.5% (by wt.)	16 220	0.67
1.0	16 220	0.653
2.5	16 220	—
0.5	21 625	0.53
1.0	21 625	0.527
2.5	21 625	0.49
0.5	27 033	0.46
1.0	27 033	0.43
2.5	27 033	0.34

* Pipe diameter = 0.05715 m.

three different levels of suspended solid particles (starch granules). The added solids obviously reduced the Lagrangian time scale.

Figure 2 reports the eddy diffusivity [Equation (4)] determined at different starch concentrations in the suspension. The effect of the solids is higher at low Reynolds numbers than at higher Reynolds numbers. Kada and Hanratty (1960) qualitatively observed that for large particles, effect of solids in low concentration suspensions was also not significant at high Reynolds numbers.

The intensity of concentration fluctuations was studied at the center of the test pipe. Typical results for root mean squares of concentration fluctuations are plotted against time t in Figure 3. From the slope, t_E was determined according to Equation (5). The results are reported in Table 2.

The decay of intensity of concentration fluctuations was aided somewhat by the solids. However, in the concentration range studied, the effect was not very significant. The micromixing time constant did decrease with increasing Reynolds numbers.

ACKNOWLEDGMENT

The authors wish to acknowledge the financial support of this study by the United States Department of Agriculture through a research contract No. 12-14-100-9568.

NOTATION

A = constants
a = particle diameter
C = local salt concentration
$\overline{C}$ = time average of C
c' = root mean square fluctuating concentration
c'_0 = c' at time zero
D_E = eddy diffusivity
d = pipe diameter
I_c = intensity of concentration fluctuation
L_h = Lagrangian macroscale for diffusion
$R_L(h)$ = Lagrangian correlation coefficient

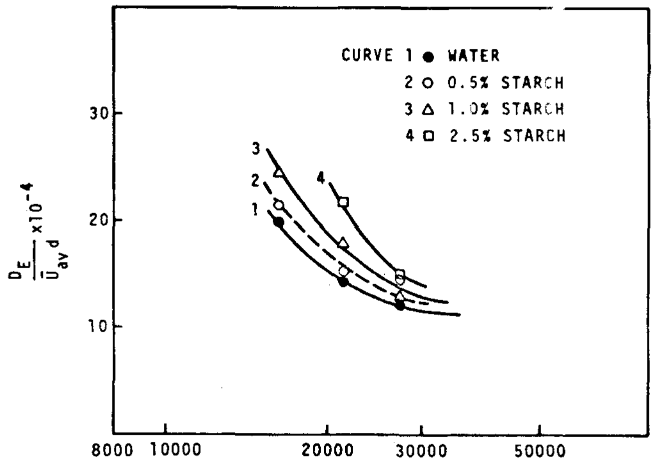

Fig. 2. Radial eddy diffusivity in starch suspensions.

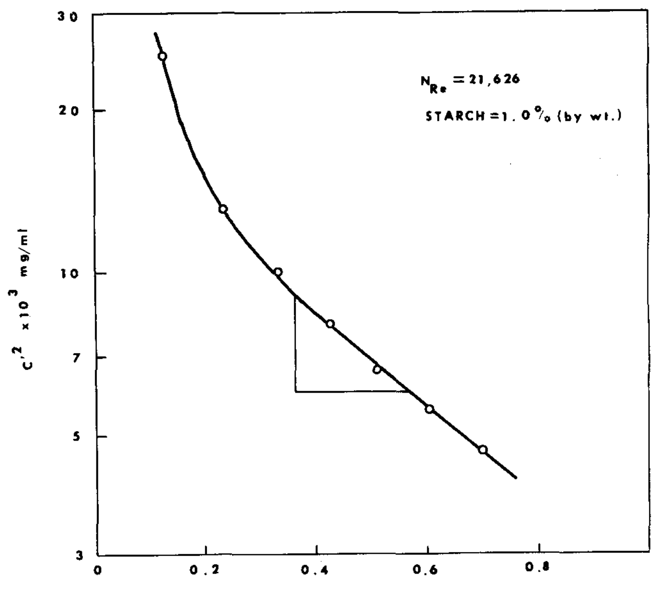

Fig. 3. Intensity of concentration fluctuations.

r = distance from pipe center
t = time
t_E = micromixing time constant
$\overline{U}$ = time average local velocity
u' = root mean square of u
x = displacement in the lateral direction of the pipe

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Manuscript received July 21, 1975; revision received October 31, and accepted November 4, 1975.